

$(g - 2)_\mu$ and supersymmetry: status and prospects

Dominik Stöckinger ^a

Department for Physics & Astronomy, University of Glasgow

Abstract. The experimental determination of the muon magnetic moment and its theoretical prediction within the Standard Model and the MSSM are reviewed. A 3σ deviation between experiment and Standard Model prediction has been established, and supersymmetry could provide a natural explanation of this deviation. Possible future improvements and the case for a new experiment are discussed.

PACS. 13.40.Em Electric and magnetic moments – 14.60.Ef Muons – 12.60.Jv Supersymmetric models

1 Introduction

For decades, the main arguments in favour of supersymmetry at or below the TeV-scale have been of a theoretical nature and have been related to e.g. naturalness in the Higgs sector or unification of gauge couplings. Now, the muon magnetic moment $a_\mu = (g - 2)_\mu/2$ has developed into one of the strongest and most robust observational indications for the existence of supersymmetry at or below the TeV-scale. Independent of theoretical naturalness or unification arguments, a 3σ deviation between the experimental and Standard Model (SM) theory value of a_μ has been established, and this deviation can be well explained by TeV-scale supersymmetry but is very hard to accommodate within many other scenarios for physics beyond the SM. In these proceedings¹ we briefly review the current status of a_μ within the SM and the minimal supersymmetric Standard Model (MSSM) and we discuss possible future improvements.

2 Deviation between the experimental value and the SM theory prediction

The muon magnetic moment has been measured at the recent E821 experiment at Brookhaven. The final result of this experiment reads [3]

$$a_\mu^{\text{exp}} = 11\,659\,208.0(6.3) \times 10^{-10}. \quad (1)$$

The success of this experiment has inspired tremendous progress also on the SM theory evaluation of a_μ , particularly on the hadronic vacuum polarization contributions and hadronic light-by-light contributions, the two contributions with the by far largest theory uncertainties.

The hadronic light-by-light contributions are tiny but important at the current level of precision. They cannot be evaluated from first principles. In the 1990's these contributions have been evaluated by two groups [4, 5]. Since then, major progress has been made in two directions: first a sign error in these calculations was uncovered in [6] (and confirmed by the original authors), the correction of which shifted the SM theory prediction by more than $+16 \times 10^{-10}$. Second, new short-distance constraints on the relevant light-by-light correlator were studied and incorporated into the computation in [7], which again shifted the result by about 5×10^{-10} . The recent developments are reviewed in more detail in [8, 9, 10], and current estimates for the hadronic light-by-light contributions vary between

$$a_\mu^{\text{LbL}} = 10.0(3.9)[11] \dots 13.6(2.5)[7]. \quad (2)$$

The hadronic vacuum polarization contributions are currently the dominant source of the SM theory uncertainty. Via the optical theorem, they can be related to the cross section for $e^+e^- \rightarrow \text{hadrons}$, which can be measured. The recent progress is due to refined ways to combine existing experimental data on $e^+e^- \rightarrow \text{hadrons}$, obtained from different experiments and for different energies, and to improved measurements of $e^+e^- \rightarrow \text{hadrons}$. In the last 10 years, new results on this cross section have become available from BES-II [12], CMD-2 [13], and most recently from SND [14] and CMD-2 [15], both in Novosibirsk, and from KLOE [16] and BaBar. The KLOE measurement is particularly interesting since it is the first one using radiative return measurements. Three major groups [17, 18, 11] have presented updated evaluations that incorporate the latest measurements², with results in the

^a Email: d.stockinger@physics.gla.ac.uk

¹ These proceedings are based on [1, 2].

² They differ e.g. in the way they incorporate the KLOE data.

range

$$a_\mu^{\text{vac.pol.}} = 689.4(4.6)[18] \dots 692.1(5.6)[11]. \quad (3)$$

In principle, part of the $e^+e^- \rightarrow \text{hadrons}$ cross section could be obtained in an alternative way from hadronic τ decays [19]. This was particularly useful when the e^+e^- data was rather imprecise and dominated by only the CMD-2 data. Now, several e^+e^- data sets are available, and a disagreement between e^+e^- based and τ data based analyses of $a_\mu^{\text{vac.pol.}}$ has led most groups to a preference of the theoretically cleaner e^+e^- based analyses (see e.g. the discussions in [17,9]).

After the most recent progress the SM theory prediction for a_μ has reached a very mature state. The full prediction is obtained by adding the QED and electroweak to the hadronic contributions. The review [9] obtains

$$a_\mu^{\text{SM}} = 11\,659\,178.5(6.1) \times 10^{-10} \quad (4)$$

and thus

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 29.5(8.8) \times 10^{-10}, \quad (5)$$

a 3.4σ deviation! The results obtained in [17,18,11] differ slightly but all obtain deviations of more than 3σ . Therefore a 3σ deviation between the experimental and the SM theory value of a_μ has been firmly established.

3 Muon magnetic moment and supersymmetry

If the observed 3σ deviation is not due to an error or a statistical fluctuation, where could it come from? If supersymmetry (SUSY) exists, the superpartner particles would give rise to a contribution to a_μ of approximately

$$a_\mu^{\text{SUSY}} \approx 13 \times 10^{-10} \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan\beta \text{sign}(\mu), \quad (6)$$

where M_{SUSY} denotes the common superpartner mass scale, $\tan\beta$ the ratio of the two Higgs vacuum expectation values, and μ the Higgsino mass parameter. Hence, supersymmetry could easily be the origin of the observed deviation of 29.5×10^{-10} , e.g. for SUSY masses of roughly 200 GeV and $\tan\beta \sim 10$ or SUSY masses of 500 GeV and $\tan\beta \sim 50$.

Although this result is very well known and has been stressed many times, see e.g. [20], it is quite non-trivial and singles out supersymmetry among many extensions of the SM. One should note that the deviation of 29.5×10^{-10} is almost twice as high as the SM electroweak contributions, i.e. diagrams with W , Z , Higgs exchange etc.,

$$a_\mu^{\text{SM EW}} = 15.4(0.2) \times 10^{-10}. \quad (7)$$

Likewise, a generic extension of the SM with weakly interacting particles and characteristic mass scale M_{BSM}

will be suppressed by $(M_W/M_{\text{BSM}})^2$ and lead to contributions of the order

$$a_\mu^{\text{BSM generic}} \propto \left(\frac{300 \text{ GeV}}{M_{\text{BSM}}} \right)^2 \times 10^{-10}, \quad (8)$$

which is far too small except for very small M_{BSM} , which is typically already ruled out.

3.1 $\tan\beta$ enhancement

Supersymmetry has two advantages compared to such generic extensions of the SM: First, masses for the relevant supersymmetric particles, mainly smuons and charginos as small as $M_{\text{SUSY}} \sim 100 \text{ GeV}$ are still experimentally allowed. Second, the parameter $\tan\beta$ can provide an enhancement by a factor of up to about 50.

The $\tan\beta \text{sign}(\mu)$ behaviour can be easily explained on a diagrammatic level. Each diagram contributing to a_μ must contain a chirality flip between a left- and a right-handed (s)muon. The $\tan\beta$ -enhancement arises in diagrams where the necessary chirality flip occurs at a muon Yukawa coupling, either to a Higgsino or Higgs boson, because this coupling is enhanced by $1/\cos\beta \approx \tan\beta$ compared to its SM value. The μ -parameter mediates the transition between the two Higgs/Higgsino doublets $H_{1,2}$, and this transition enhances diagrams because only H_1 couples to muons while H_2 has the larger vacuum expectation value, $v_2/v_1 = \tan\beta$. Therefore, all $\tan\beta$ -enhanced terms are also proportional to $\text{sign}(\mu)$. This behaviour is not restricted to the one-loop level but repeats itself in higher orders.

3.2 Status of the MSSM prediction for the muon magnetic moment

The fact that supersymmetry is potentially the origin of the observed 3σ deviation justifies a precise analysis of the prediction for a_μ within the MSSM (for a review see [1]). The MSSM prediction is given by the SM prediction plus the genuine SUSY contributions, arising from diagrams with SUSY particle loops.

The SUSY one-loop contributions consist of diagrams with chargino/sneutrino or neutralino/smuon loops. These diagrams have been known for a long time [21]. The full expression is not repeated here. The approximation (6) can serve as a guideline. The mass parameters governing the one-loop SUSY contributions are mainly the left-handed smuon mass $m_{L,\tilde{\mu}}$ and the gaugino mass M_2 , while μ and the right-handed smuon mass $m_{R,\tilde{\mu}}$ have a smaller influence. If all mass parameters are equal to M_{SUSY} , (6) is an excellent approximation. If there are mass splittings, (6) still provides a reasonable estimate if M_{SUSY} is identified with a value between $m_{L,\tilde{\mu}}$ and M_2 . Contrary to the other mass parameters, increasing μ can lead to enhancements, e.g. if $m_{L,\tilde{\mu}} \approx m_{R,\tilde{\mu}} \approx M_2 \ll \mu$ due to a diagram with bino exchange, which is directly $\propto \mu$.

At the two-loop level two kinds of SUSY contributions are known. QED-logarithms $\log(M_{\text{SUSY}}/m_\mu)$

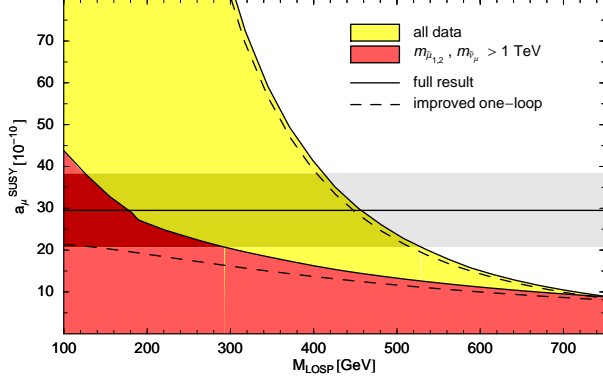


Fig. 1. Possible values of a_μ^{SUSY} as a function of the mass of the lightest observable SUSY particle M_{LOSP} , from a scan of the MSSM parameter space and for $\tan\beta = 50$. The light yellow region corresponds to all data points; the red region corresponds to points with smuons and sneutrinos that are heavier than 1 TeV. The deviation (5) is also indicated.

arising from SUSY one-loop diagrams with additional photon exchange have been evaluated in [22] and amount to $-7\% \dots -9\%$ of the one-loop contributions. Two-loop diagrams involving closed loops of either sfermions (stops, sbottoms, etc) or charginos/neutralinos have been evaluated in [23]. They amount to about 2% of the one-loop contributions if all SUSY masses are degenerate but can be much larger, e.g. if smuon masses are very heavy but stops and/or charginos and Higgs bosons are light.

A very important question regards the remaining theory error of the SUSY contributions to a_μ . This theory error arises from unknown two-loop and higher order contributions. It has been estimated in [1] to

$$\delta a_\mu^{\text{SUSY}}(\text{unknown}) = 0.02 a_\mu^{\text{SUSY,1L}} + 2.5 \times 10^{-10}, \quad (9)$$

which is smaller than the current SM theory error and the experimental uncertainty.

3.3 Implications on SUSY phenomenology

Fig. 1 summarizes the current status of a_μ and SUSY. A scan of the MSSM parameter space has been performed (for $\tan\beta = 50$ and taking into account experimental constraints from e.g. Higgs searches and b -physics; for further details see [1]), and the resulting values for a_μ^{SUSY} , including all known one- and two-loop contributions, are plotted as a function of the mass of the lightest observable SUSY particle. Fig. 1 confirms again that SUSY can easily explain the observed deviation if M_{LOSP} is below about 600 GeV.

Apart from a_μ , significant information on SUSY parameters can be inferred from the measured dark matter density, if it is assumed to consist of the stable, lightest SUSY particle. The two observables tend to constrain orthogonal directions in the multi-dimensional SUSY parameter space and are thus complementary. Several recent comprehensive studies [24, 25, 26, 27] have shown that the MSSM is able to simultaneously accommodate all existing data from a_μ , dark matter, b -physics and electroweak precision observables. This is even possible, in spite of some slight

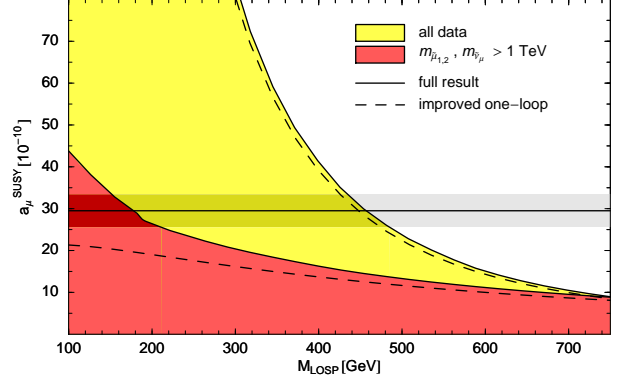


Fig. 2. As fig. 1 but showing the future precision of the deviation (10).

tensions, in the constrained MSSM (CMSSM), a model with only 4 input parameters. One result of these studies is that rather low SUSY masses are preferred as a consequence of the a_μ deviation.

4 Future prospects and the case for a new experiment

The present 3σ deviation is one of the strongest observational hints for the existence of supersymmetry at or below the TeV-scale. However, although the deviation is tantalizing it is not quite large enough to be regarded as a proof of physics beyond the SM. Fortunately, there are good prospects that the current uncertainty of 8.8×10^{-10} of the deviation (5) can be reduced significantly in the near future.

The current theory error of 6.1×10^{-10} of the SM prediction (4) will soon decrease due to currently ongoing more precise determinations of the $e^+e^- \rightarrow$ hadrons cross section. Both KLOE and BaBar will soon release data on the most important $\pi\pi$ channel using radiative return measurements. If these data are in agreement with the Novosibirsk data, they will not only reduce the error but also significantly increase our confidence in the e^+e^- data. The new data will immediately improve our knowledge of the hadronic vacuum polarization contributions to a_μ , which currently are the dominant source of error.

The second most important source of theory error are the hadronic light-by-light contributions. These are notoriously difficult to evaluate, but they have moved into the centre of attention, and several groups are currently investigating these contributions, using both established and novel approaches. A determination of these contributions with a relative accuracy of about 15% seems possible. In combination, a reduction of the theory error of the SM prediction down to $(3 \dots 5) \times 10^{-10}$ within the next few years seems likely.

The tantalizing status of the current deviation, together with the prospect for an improvement of the SM theory prediction, highlights the need for and the potential of a new, better experimental measurement of a_μ [2]. A corresponding experiment, E969 at Brookhaven [28], with the goal of a final uncertainty of 2.5×10^{-10} , has been proposed and received scientific approval at Brookhaven.

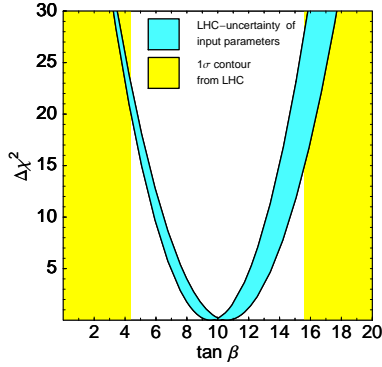


Fig. 3. Determination of $\tan\beta$ from LHC [27] (yellow region) and from a_μ (blue band), assuming (10).

In an optimistic scenario, where the theory error is reduced to 3×10^{-10} and the magnitude of the deviation between SM theory and experiment remains the same, this new measurement would lead to

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}(\text{future}) = 29.5(3.9) \times 10^{-10}. \quad (10)$$

This more than 7σ deviation would dramatically sharpen the case for new physics. The impact it would have on SUSY phenomenology is illustrated in figs. 2, 3 [2]. Fig. 2 shows the same scan of the possible SUSY contributions to a_μ as in fig. 1, versus the future deviation. The precision of (10) would lead to strong upper and lower mass bounds on SUSY particles which could complement mass measurements from LHC.

Fig. 3 illustrates how a_μ might complement even comprehensive LHC measurements. The analysis in [29,27] shows that using a global fit of the MSSM to LHC data one can determine SUSY masses rather precisely but the parameter $\tan\beta$ rather poorly. If the benchmark point SPS1a [30] is realized, the LHC-analysis of [29] yields $\tan\beta = 10.22 \pm 9.1$, the improved analysis of [27] yields $\tan\beta = 10 \pm 4.5$. Since a_μ^{SUSY} is directly proportional to $\tan\beta$, a precise determination as in (10) would provide an invaluable complement to LHC in the determination of $\tan\beta$. Fig. 3 shows the value of $\Delta\chi^2 = (a_\mu^{\text{SUSY+SM}} - a_\mu^{\text{exp}})^2 / (3.9 \times 10^{-10})^2$ as a function of $\tan\beta$. In a_μ^{SUSY} all parameters except for $\tan\beta$ have been fixed to the SPS1a values, which are accessible well at the LHC.

5 Conclusions

A tantalizing deviation of more than 3σ between the SM theory prediction and the experimental value of a_μ has been established. Supersymmetry with rather light masses and moderate to large $\tan\beta$ could easily be the origin of this deviation. The near future is very promising if the proposed E969 experiment [28] is realized. The SM theory uncertainty will soon further decrease and a new experiment could push the significance of the deviation up to more than 7σ .

Acknowledgments: It is a pleasure to thank the organizers of SUSY07 for this enjoyable conference.

References

1. D. Stöckinger, J. Phys. G **34** (2007) R45.
2. D. W. Hertzog, J. P. Miller, E. de Rafael, B. Lee Roberts and D. Stöckinger, arXiv:0705.4617 [hep-ph].
3. G. W. Bennett [Muon Collaboration], Phys. Rev. D **73** (2006) 072003.
4. J. Bijnens, E. Pallante and J. Prades, Nucl. Phys. B **474** (1996) 379.
5. M. Hayakawa, T. Kinoshita and A. I. Sanda, Phys. Rev. Lett. **75** (1995) 790; M. Hayakawa and T. Kinoshita, Phys. Rev. D **57** (1998) 465 [Erratum-ibid. D **66** (2002) 019902].
6. M. Knecht, A. Nyffeler, M. Perrottet and E. De Rafael, Phys. Rev. Lett. **88** (2002) 071802.
7. K. Melnikov and A. Vainshtein, Phys. Rev. D **70** (2004) 113006.
8. J. Bijnens and J. Prades, Mod. Phys. Lett. A **22** (2007) 767.
9. J. P. Miller, E. de Rafael and B. L. Roberts, Rept. Prog. Phys. **70** (2007) 795.
10. A. Czarnecki, *this conference*.
11. F. Jegerlehner, arXiv:hep-ph/0703125.
12. J. Z. Bai *et al.* [BES Collaboration], Phys. Rev. Lett. **84** (2000) 594; Phys. Rev. Lett. **88** (2002) 101802.
13. R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **578** (2004) 285; Phys. Lett. B **527** (2002) 161.
14. M. N. Achasov *et al.*, J. Exp. Theor. Phys. **103** (2006) 380 [Zh. Eksp. Teor. Fiz. **130** (2006) 437].
15. V. M. Aulchenko *et al.* [CMD-2 Collaboration], JETP Lett. **82** (2005) 743; R. R. Akhmetshin *et al.*, JETP Lett. **84** (2006) 413; Phys. Lett. B **648** (2007) 28.
16. A. Aloisio *et al.* [KLOE Collaboration], Phys. Lett. B **606** (2005) 12.
17. M. Davier, Nucl. Phys. Proc. Suppl. **169** (2007) 288.
18. K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, Phys. Lett. B **649** (2007) 173.
19. R. Alemany, M. Davier and A. Hocker, Eur. Phys. J. C **2** (1998) 123.
20. A. Czarnecki and W. J. Marciano, Phys. Rev. D **64** (2001) 013014.
21. J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D **49**, 366 (1994); U. Chattopadhyay and P. Nath, Phys. Rev. D **53**, 1648 (1996); T. Moroi, Phys. Rev. D **53** (1996) 6565 [Erratum-ibid. **56** (1997) 4424].
22. G. Degrossi and G. F. Giudice, Phys. Rev. D **58** (1998) 053007.
23. S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B **690** (2004) 62; Nucl. Phys. B **699** (2004) 103.
24. R. R. de Austri, R. Trotta and L. Roszkowski, JHEP **0605** (2006) 002.
25. B. C. Allanach, K. Cranmer, C. G. Lester and A. M. Weber, arXiv:0705.0487 [hep-ph].
26. J. R. Ellis, S. Heinemeyer, K. A. Olive, A. M. Weber and G. Weiglein, arXiv:0706.0652 [hep-ph].
27. R. Lafaye, T. Plehn, M. Rauch and D. Zerwas, arXiv:0709.3985 [hep-ph].
28. <http://g2pc1.bu.edu/roberts/Proposal969.pdf>
29. R. Lafaye, T. Plehn and D. Zerwas, Contribution to LHC-LC Study Group, G. Weiglein, et al. [hep-ph/0404282].
30. B. C. Allanach *et al.*, in Eur. Phys. J. C **25** (2002) 113 [eConf **C010630** (2001) P125].